Optimization of Novel High-Power Millimeter-Wave TM_{01}–TE_{11} Mode Converters

Shiwen Yang and Hongfu Li

Abstract—**In this paper, a numerical study of direct TM_{01}–TE_{11} mode converters in highly overmoded, bent smooth, circular waveguides is presented for high-power millimeter waves. The various shapes of bent waveguides are elaborately chosen, and their optimized geometrical dimensions have been achieved with a general optimization code employing the coupled mode theory. The mode converters designed can have high conversion efficiencies over 98% and wide bandwidths of 28%.**

Index Terms—**Bent waveguide, mode coupling, mode converter.**

I. Introduction

Some high-power microwave (HPM) sources, such as the virtual cathode oscillator (VCO) and the relativistic backward wave oscillator (BWO), usually launch one or mixtures of several circular TM_{0n} modes. These modes are unsuitable for their succeeding applications, mainly because of their undesirable conical radiation patterns. In many applications, the circular TE_{11} mode is needed, for it has a definite polarization and a radiation pattern which is similar to that of the "Gaussian-like" HE_{11} hybrid mode in a circumferentially corrugated waveguide. If necessary, the TE_{11} mode can be efficiently transformed to the HE_{11} mode in a very short converter length [1]. Therefore, one needs a kind of TM_{0n}–TE_{11} mode converter with high conversion efficiency, high power capacity, proper bandwidth, and as short as possible.

Since gyrotrons have been used successfully for plasma heating, many studies on TE_{0n}–TE_{11} conversion have been carried out [2]–[4]. But there is relatively little study on TM_{0n}–TE_{11} mode converters. The TM_{0n}–TE_{11} transformation is fulfilled generally by a two-step process: TM_{0n}–TM_{01}–TE_{11}. The TM_{0n}–TM_{01} mode converter may be easily realized [5]. The TM_{01}–TE_{11} mode converter, however, cannot be directly realized by adopting the conventional periodically bent waveguide mode converters [2], [3] in a short converter length, due to the long beat wavelength between TM_{01} and TE_{11} at high frequencies and large waveguide diameters. The authors have used an intermediate mode TM_{11} to realize an 8-mm TM_{01}–TM_{11}–TE_{11} mode converter [6], which is shorter than the conventional, periodically bent, waveguide mode converters. This mode converter allows an arbitrary choice and fast change of the polarization plane, but only an overall conversion efficiency of $\eta = 94.7\%$ and an overall bandwidth of 2.06% (for $\eta \geq 90\%$) are obtained. This is apparently not satisfactory for some applications, especially at high power levels and high frequencies.

Based on the systematic study on mode coupling [7], this paper reports another method to realize the direct high efficiency TM_{01}–TE_{11} mode converters, which are formed with smooth-walled waveguides bent in one plane in very short converter lengths. In the case of a degenerate mode TE_{01}–TM_{11} mode converter, a waveguide bend with appropriate constant curvature and converter length is enough to fulfill the complete TE_{01}–TM_{11} mode conversion [3], [4]. However, the TM_{01}–TE_{11} mode converter cannot be easily realized by a waveguide bend with constant curvature or any other single curvature distribution (the curvature does not change its sign). The authors' calculations show that it's necessary to change the signs of waveguide curvature at least one time to achieve the high efficiency TM_{01}–TE_{11} conversion. By optimization, the authors present five kinds of bent TM_{01}–TE_{11} mode converters with different structures. Each of the five mode converters has a high conversion efficiency of about 98%. Furthermore, it has much wider bandwidth of about 28%. Many factors, including the spurious modes, backward waves, phase-rematching techniques, and ohmic losses are taken into account in the authors' calculations. Although ellipticity is inevitably produced in the waveguide in the process of waveguide bending, the influence of ellipticity produced by precise manufacturing on the conversion efficiency turns out to be negligible [4]. So, the influence of ellipticity is not included in the authors' calculations.

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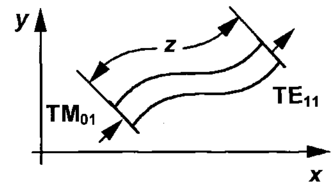

Fig. 1. Geometry scheme of the bent TM_{01}–TE_{11} mode converter.

II. General Considerations

Mode conversion due to curvature in circular corrugated or smooth waveguides has been systematically studied in [7]. The (mn)th mode converts to $(m'n')$th mode only in condition that $|m - m'| = 1$, and the first-order coupled differential equations for this kind of axis-curved waveguide mode converters are

$$\frac{dA^+_{m'n'}}{dz} = -\gamma_{m'n'} - j\sum_{mn}\left[C^+_{(m'n')(mn)}A^+_{mn} + C^-_{(m'n')(mn)}A^-_{mn}\right] \tag{1}$$

$$\frac{dA^-_{m'n'}}{dz} = \gamma_{m'n'} + j\sum_{mn}\left[C^+_{(m'n')(mn)}A^-_{mn} + C^-_{(m'n')(mn)}A^+_{mn}\right] \tag{2}$$

where A^+_{mn} and A^-_{mn} represent the complex amplitudes of forward and backward waves for the (mn)th mode, respectively, $\gamma_{mn} = \alpha_{mn} + j\beta_{mn}$ is the propagation constant of the (mn)th mode, α_{mn} is the attenuation constant, and β_{mn} is the axial propagation constant. Since the bent waveguide curvature cannot be regarded as the perturbation of a straight waveguide, the curve integral elements dz in (1) and (2) should be along the waveguide axis. If the waveguide axis coordinate in the x–y plane of the waveguide bend, which is shown in Fig. 1, can be expressed as $y = f(x)$, then the curve integral element dz in (1) and (2) should be

$$dz = \sqrt{1 + (df(x)/dx)^2}\,dx. \tag{3}$$

In (1) and (2) $C^+_{(m'n')(mn)}$ are the coupling coefficients between the (mn)th mode and the $(m'n')$th mode whose directions of propagation are the same, while $C^-_{(m'n')(mn)}$ are those between two corresponding modes whose directions of propagation are opposite. Their general and explicit formulas are given in [7]. For the TM_{01}–TE_{11} mode converters, the TM_{11} is the most dangerous spurious mode near the beginning where TM_{01} has a higher power level, while near the output where TE_{11} has a higher power level,

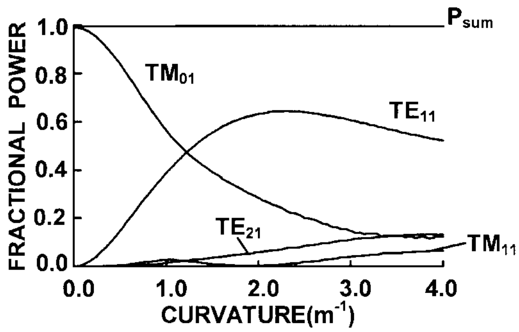

Fig. 2. Fractional power levels versus waveguide curvatures, when TE_{11} reaches its first maximum, in a simple bent TM_{01}–TE_{11} mode converter with constant curvature ($f_0 = 35$ GHz, $a_0 = 13.6$ mm).

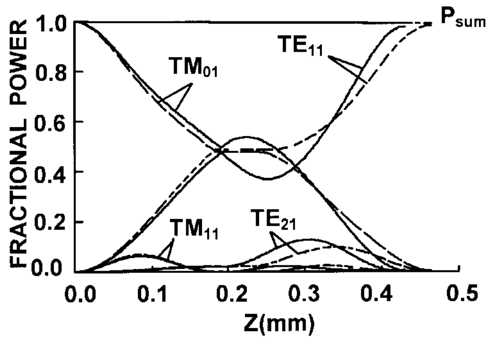

Fig. 3. Calculated fractional power distributions for the TM_{01}–TE_{11} mode converter of Case (a) (solid line) and Case (b) (dashed) line in Table I.

TE_{21} becomes the most dangerous spurious mode. Fig. 2 illustrates the fractional power level in each mode as a function of the constant waveguide curvature, when TE_{11} reaches its first maximum in a waveguide with constant curvature (the frequency $f_0 = 35$ GHz and the waveguide radius $a_0 = 13.6$ mm). As can be seen, along with the increasing of waveguide curvature, the couplings to TM_{11} and TE_{21} will ultimately hinder the steady increasing of the TE_{11} mode level.

III. Computational Results

Six coupled modes included in the authors' calculations are as follows: TM_{01}, TE_{11}, TM_{11}, TE_{21}, TE_{01}, and TM_{12}. The influence of other spurious modes seem to be negligible according to the calculation results. Ohmic losses are included in all the calculations. The authors assume the amplitude of TM_{01} at the input to be unit, the boundary conditions for the TM_{01}–TE_{11} mode converter are [6]

$$A^+_{mn}|_{z=0} = [(1,0),(0,0),\ldots,(0,0)]^T \tag{4}$$

$$A^-_{mn}|_{z=L} = [(0,0),(0,0),\ldots,(0,0)]^T \tag{5}$$

where the first element of the six-dimensional (6–D) vectors is for TM_{01}, the second is for TE_{11}, and so on. Also, the authors have introduced a transmission matrix method in [6] to simplify the procedure of solving the boundary value problem, which is composed of (1), (2), (4), and (5), by using the boundary conditions' simplicity. The authors wrote a general optimization program, in which the conversion efficiencies are chosen as maximum objective functions, and the geometrical characteristics of the mode converters are chosen as free parameters to be optimized. The authors tested the program by calculating the examples for TE_{01}–TM_{11} mode converters [3], [4]. The authors' results are in excellent agreement with that of [3], [4]. Through optimization, the authors present five kinds of high efficiency bent TM_{01}–TE_{11} mode converters with different structures.

TABLE I
Optimized Characteristic Results for TM_{01}–TE_{11} Mode Converters with Segmental Constant Curvatures ($f_0 = 35$ GHz, $a_0 = 13.6$ mm)

Case	Without phase delay section (a)	With phase delay section (b)
Bend angle: θ_I	14.58°	14.77°
θ_{II}	21.72°	15.55°
Arc length: L_I	0.1906m	0.1836m
L_{II}	0.2387m	0.2224m
Phase delay section length: L_{III}	—	0.0577m
Total converter length	0.4293m	0.4636m
Output Power levels: TM_{01}	0.0019	0.0010
TE_{11}	0.9809(η)	0.9846(η)
TM_{11}	0.0021	0.0034
TE_{21}	0.0055	0.0041
TE_{01}	0.0036	0.0003
Power transmission efficiency: P_{sum}	0.9940	0.9939
Bandwidth factor (η≥90%): Δf/f₀	27.4%	28.3%

A. TM_{01}–TE_{11} Mode Converters Formed by Combinations of Two Bent Waveguides with Constant Curvatures or by Combinations of Two Constantly Curved Waveguides and a Straight Waveguide

The TE_{01}–TM_{11} mode converter may be realized by two waveguide bends with constant curvatures, whose signs should be opposite. The two sections are linked up at the tangent points of the two arcs, while the input and output waveguide are also tangent to the arcs so that the junctures will not produce additional coupling or reflection. Also, the converter may be improved with a phase-delay section of straight waveguide placed tangentially between the two waveguide arcs. The optimized geometrical dimensions, as well as their output power levels and bandwidth factors for both of the two kinds of converters, are shown in Table I Case (a) and Case (b), respectively, for $f_0 = 35$ GHz and $a_0 = 13.6$ mm. The calculated backward wave power levels (mainly the TM_{01} reflection) at the input are generally below -80 dB. Fig. 3 shows the fractional power distributions of forward waves and total transmitted power efficiency $(P_{\rm sum})$ as functions of z along the converters' axes.

B. TM_{01}-TE_{11} Mode Converters Formed by Bent Waveguides with Continuously Nonconstant Curvatures

The authors chose three kinds of TM_{01}–TE_{11} mode converters with nonconstant curvature distributions, whose waveguide axis coordinates in the x–y plane take the forms as follows:

1) period of sinusoidal distribution

$$y = \varepsilon_1 \cos\frac{2\pi}{W_1}x - \varepsilon_2 \sin\frac{2\pi}{W_2}x - \varepsilon_3 \sin\frac{2\pi}{W_3}x \tag{6}$$

2) cubic parabola distribution

$$y = \varepsilon_1 x^3 - \varepsilon_2 \sin\frac{2\pi}{W_2}x - \varepsilon_3 \sin\frac{2\pi}{W_3}x \tag{7}$$

3) Gaussian distribution

$$y = \varepsilon_1 e^{-\delta x^2} - \varepsilon_2 \sin\frac{2\pi}{W_2}x - \varepsilon_3 \sin\frac{2\pi}{W_3}x. \tag{8}$$

The period of the sinusoidal mode converter corresponds approximately to the beat wavelength between TM_{01} and TE_{11}, while the

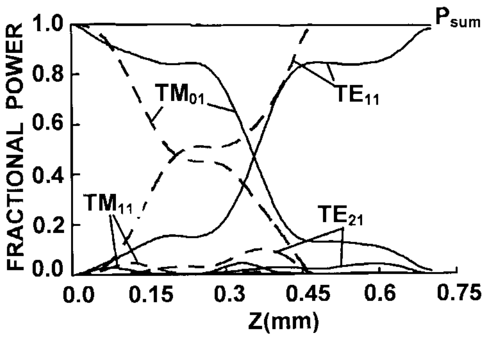

Fig. 4. Calculated fractional power distributions for the TM_{01}–TE_{11} mode converter of Case (a) (solid line) and Case (c) (dashed line) in Table II.

TABLE II
OPTIMIZED CHARACTERISTIC RESULTS FOR TM_{01}–TE_{11} MODE CONVERTERS FORMED BY NONCONSTANTLY BENT WAVEGUIDES ($f_0 = 35$ GHz, $a_0 = 13.6$ mm)

Case	Sinusoidal (a)	Cub. Parabo. (b)	Gaussian (c)
Axis coordinate:	Eq. (6)	Eq. (7)	Eq. (8)
Geometrical characteristics:	ε_1=0.0162m	ε_1=1.5810m^{-2}	ε_1=0.1000m
	W_1=0.6971m		δ=14.085m^{-2}
	ε_2=0.0002m	ε_2=-0.0001m	ε_2=-0.0004m
	W_2=0.1800m	W_2=0.1595m	W_2=0.2067m
	ε_3=0.0007m	ε_3=0.0011m	ε_3=-0.0015m
	W_3=0.2781m	W_3=0.2796m	W_3=0.2798m
Total converter length:	0.7008m	0.4749m	0.4715m
Output power levels: TM_{01}	0.0125	0.0010	0.0002
TE_{11}	0.9756(η)	0.9857(η)	0.9887(η)
TM_{11}	0.0001	0.0025	0.0004
TE_{21}	0.0005	0.0016	0.0003
TE_{01}	0.0012	0.0027	0.0040
Power transmission efficiency: P_{sum}	0.9900	0.9935	0.9936
Bandwidth factor ($\eta \geq 90\%$): $\Delta f/f_0$	21.1%	28.2%	27.1%

cubic parabola mode converter has an antisymmetry about its center ($x = 0$), and the Gaussian mode converter takes a segment of the Gaussian curve's left side ($x < 0$). The two additional, continuous phase-rematching small perturbations ε_2 and ε_3 are used to suppress the TM_{11} and TE_{21}, respectively. All the calculated results of the three TM_{01}–TE_{11} mode converters for $f_0 = 35$ GHz and $a_0 = 13.6$ mm, corresponding to Cases (a), (b), and (c) are summarized in Table II. Fig. 4 demonstrates the normalized power distributions for Cases (a) and (c) in Table II. The distributions for Case (b) are similar to that of Case (c) and are not plotted in Fig. 4.

IV. CONCLUSION

Direct short TM_{01}–TE_{11} mode converters with high conversion efficiencies and large bandwidth factors may be realized by bent waveguides with elaborately chosen shapes and optimized geometrical dimensions. Five examples of TM_{01}–TE_{11} mode converters with different structures for $f_0 = 35$ GHz and waveguide radius $a_0 = 27.2$ mm are presented, with each one having its own features. All of the five mode converters can have high conversion efficiencies of $\eta > 97.5\%$ and large bandwidth factors (for $\eta \geq 90\%$) of greater than 21%.

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Characteristic Impedance of a Rectangular Double-Ridged TEM Line

Khona Garb and Raphael Kastner

***Abstract*— The characteristic impedance of a TEM transmission line, shaped as a double-ridged rectangular coaxial line, is analyzed in this paper as the customary transversal static problem. This type of transmission line is useful, for example, as a part of a cascaded transition between a double-ridged waveguide and a coaxial line. The solution of the transversal problem is achieved by dividing the cross-sectional region into distinct, separable regions, each one being characterized by a closed-form Green's functions relating the flux function to the electric field. Surface-type integral equations are then formulated over the boundaries between the regions. Solution of these equations via the method of moments (MoM's) using the Galerkin choice yields the results for the characteristic impedance as a function of cross-sectional dimensions. Convergence of the solution is also studied.**

***Index Terms*— Characteristic impedance, Galerkin method, ridged TEM line.**

I. INTRODUCTION

Transmission lines operating in the TEM regime are widely used in microwaves circuits. In many cases, TEM and non-TEM devices need to be connected in a cascaded configuration, requiring a transition between the two types of lines. Such a transition would possess some of the geometrical features of the two lines at both ends. In the case treated in this paper, a transition between a coaxial TEM line and a double-ridged waveguide has been devised. Several rectangular

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